NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA
(An Autonomous Institute Affiliated to AKTU, Lucknow)
B.Tech
SEM: II - THEORY EXAMINATION (2024 - 2025)
Subject: Linear Algebra

Time: 3 Hours   Max. Marks: 100

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*
*1. This Question paper comprises of **three Sections -A, B, & C.** It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. Maximum marks for each question are indicated on right -hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.
6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION-A   20

1. Attempt all parts:-

1-a.   If a matrix A is symmetric as well as skew-symmetric, then: (CO1, K1)   1

   (a) A is diagonal matrix
   (b)   A is a unit matrix
   (c)   A is a triangular matrix
   (d)   A is a null matrix

1-b.   1

   If the matrix $\begin{bmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$ is singular, then λ equals (CO1,K2)

   (a)   -2
   (b)   2
   (c)   4
   (d)   -4

1-c.   If the rank of a matrix A is 2, then the rank of A' is (CO2, K2)   1

   (a)   *3*
   (b)   *2*
   (c)   8
   (d)   *16*

1-d.   If the set of r n-vectors are dependent, then values of all scalars be (CO2,K2)   1

(a)   0

(b)   1

(c)   2

(d)   None of these

1-e.   Which of the following is false? (CO3,K2)   1

(a)   Every subspace of a vector space is itself a vector space.

(b)   Every vector space is a subspace of itself.

(c)   The intersection of any two subspaces of a vector space V is a subspace of V.

(d)   The union of any two subspaces of a vector space V is a subspace of V

1-f.   In the vector space V if $a + b = b + a$;where a, b $\in$ V is called (CO3,K1)   1

(a)   Associativity

(b)   Additive inverse

(c)   Commutative

(d)   None of these

1-g.   What should be the Eigen values for the matrix given below?   1

$$A = \begin{bmatrix} 6 & 3 \\ 4 & 5 \end{bmatrix}$$ (CO4,K3)

(a)   2 & 9

(b)   2 & 10

(c)   1 & 9

(d)   2 & 7

1-h.   What is the general form of characteristic equation of matrix? (CO4,K1)   1

(a) $|A - \lambda I| = 0$

(b) $|A - \lambda| = 0$

(c) $|A - I| = 0$

(d) $|AI - \lambda I| = 0$

1-i.   If the order of A is 4×3, the order of B is 4×5 and the order of C is 7×3, then the order of $(A^T B)^T C^T$ is (CO5,K2)   1

(a) 5×3

(b) 4×5

(c) 5×7

(d) 4×3

1-j.   In PCA, what are the principal components? (CO5,K4)   1

(a)   Features of the dataset

(b)   Eigenvalues of the covariance matrix

(c)   Eigenvectors of the covariance matrix

(d)   Data points in the dataset

2. Attempt all parts:-

2.a.   Define orthogonal matrix. (CO1,K1)   2

2.b.   If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then find the rank of AA^T. (CO2,K2)   2

2.c.   Define norm of a vector. (CO3,K2)   2

2.d.   For the matrix A, Find the sum of eigen values where $A = \begin{bmatrix} -2 & 1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. (CO4,K2)   2

2.e.   Write any two examples of singular value decomposition in machine learning. (CO5,K5)   2

SECTION-B   30

3. Answer any <u>five</u> of the following:-

3-a.   By using the properties of determinants, show that: 6
$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$. (CO1,K2)

3-b.   Find the value of x and y if: $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$. (CO1,K2)   6

3-c.   Find the rank of the matrix: $A = \begin{bmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{bmatrix}$. (CO2,K2)   6

3-d.   Solve the system of equations: $x+y+z=0, x+2y-z=0, 2x+y+3z=0$. (CO2,K3)   6

3.e.   If α and β are vectors in an inner product space then show that $\|\alpha+\beta\|^2 + \|\alpha-\beta\|^2 = 2\|\alpha\|^2 + 2\|\beta\|^2$. (CO3,K3)   6

3.f.   Find the eigen values and eigen vector of the matrix of $A = \begin{bmatrix} 3 & 0 & 0 \\ 1 & -2 & -8 \\ 0 & -5 & 1 \end{bmatrix}$. (CO4,K3)   6

3.g.   What are the applications of principal component analysis in image processing? Explain. (CO5,K4)   6

SECTION-C   50

4. Answer any <u>one</u> of the following:-

4-a.   Calculate the inverse of the   10

$$A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}. \text{ (CO1,K2)}$$

4-b. Solve the system of equations for the linear equations with 3 variables using Cramer's rule. (CO1,K2)   10

$x - z = 11$

$7x - 4y - z = -15$

$4x + 6y + 5z = -6$

5. Answer any <u>one</u> of the following:-

5-a. Show that the equations $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$ are consistent and solve them by rank method. (CO2,K3)   10

5-b. Find the rank of the matrices by reducing it to canonical form: (CO2,K3)   10

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

6. Answer any <u>one</u> of the following:-

6-a. Show that for the vectors $\alpha = (x_1, x_2)$ and $\beta = (y_1, y_2)$ from $\mathbf{R}^2$, the following defines an inner product on $\mathbf{R}^2$:   10

$$< \alpha, \beta > = x_1 y_1 - x_2 y_1 - x_1 y_2 + 2x_2 y_2. \text{ (CO3, K3)}$$

6-b. Show that the vectors $(3,0,0,0), (3,-1,1,0), (5,-1,1,3), (6,0,1,3)$ are linearly independent over R. (CO3,K3)   10

7. Answer any <u>one</u> of the following:-

7-a. Show that the mapping $T: V_2 \rightarrow V_3$ defined as $T(a,b) = (a+b, a-b, b)$ is a linear transformation from V_2 into V_3. Find the range, rank, null-space and nullity of T. (CO4,K3)   10

7-b.   10

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$$

Show that the matrix is a Unitary Matrix, where ω is a cube root of unity. (CO4,K3).

8. Answer any <u>one</u> of the following:-

8-a.   10

Find the Singular Value Decomposition of a matrix $\begin{bmatrix} -4 & -7 \\ 1 & 4 \end{bmatrix}$. (CO5,K4)

8-b. Find the covariance matrix and the principal components of the following: (CO5,K4)   10

X	2.5	0.5	2.2	1.9	3.1	2.3
Y	2.4	0.7	2.9	2.2	3.0	2.7

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